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Achieving the vertical golden proportion by optimizing the width-height ratio in the anterior attractive region

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Abstract

Achieving a pleasant smile is the most important thing for improving individual confidence and it will be decided by the contemporary aesthetic methods that target gum contouring, smile line, lip line, tooth color, and shape. Even though the Golden ratio (Φ) helps with inter tooth width proportions perfectly, we can't solely rely on it. Because the tooth's vertical dimensions must be balanced and beauty is subjective, otherwise it will end up as a gummy smile. So, this paper introduces and validates a new mathematical formula to solve the issue which helps design a balanced anterior aesthetic segment. This formula combines the "Golden ratio" and 7/3 ratio cause Golden ratio for inter tooth and 7/3 ratio for intra tooth. So, when both of these ratios apply consistently, the front teeth naturally form a pleasant smile and comply with the Golden ratio concept. By exploring Mathematical formulas with dentistry, this research innovates a detailed system where the dental professionals work to create a perfectly balanced smile. This theory also ensures that symmetry is achieved in both vertical and horizontal dimensions as well as minimizes the errors and improved outcomes.

Keywords: Golden rule, aesthetic dentistry, canine, central incisor, lateral, vertical golden proportion, teeth

Introduction

A person's well health, self-confidence, and face beauty are all greatly influenced by a lovely grin. Achieving an attractive smile in modern dentistry involves both restoring function and enhancing natural beauty. Because of this, dentists and dental technicians always strive to meet patients' expectations by creating natural, attractive smiles. Smile aesthetics analysis has typically relied on the clinician's subjective assessment. However, that is subjective and open to change. We must include mathematics into dentistry in order to fit into every category. Since mathematical ratios, especially the Golden Ratio (1:1.618), have become a crucial tool in cosmetic and restorative dentistry, mathematics will aid in this process with improved accuracy and objectivity.

Tooth selection, tooth arrangement, and restoration are the three primary phases. In each of these phases, mathematical proportions provide a methodical, evidence-based approach to manage it perfectly. The Golden radio has been doing a great job to make sure harmony and the balance between teeth, lips and facial appearance as well as granting predictable and aesthetic outcomes. But using the golden ratio alone for better outcomes seems to be a difficult option for individuals, so the dental professionals use digital smile design and CAD/CAM technologies for more accurate outcomes which have made it possible to apply golden ratio more precisely in

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clinical practice The combination of the Mathematical principles and modernized dental technology tools help dental professionals to design natural looking smiles, at the same time they are able to satisfy each patient's characteristics.

Fibonacci Magic and the Golden Rule

We can see The Fibonacci pattern all around us like in life, art and nature. It can't be a random sequence since it appears so often around the whole thing. Although it appears in everything the professionals haven't gone deeply about the Fibonacci sequence in our body.

This is how the Fibonacci sequence was formed:

It always starts with 1 and 1, then always add the two numbers which are in front.

Example: 1,1,2,3,5,8,13,21,34,55

A set of ratios can be obtained by dividing each integer by the one preceding it. These ratios approach 1.618, the Golden Ratio, which is sometimes represented by the Greek letter Phi (Φ). The ratio approaches 1.618 as the numbers increase.
Here are the first few ratios:

- $r(1) = 1 \div 1 = 1$
- $r(2) = 2 \div 1 = 2$
- $r(3) = 3 \div 2 = 1.5$
- $r(4) = 5 \div 3 = 1.67$
- $r(5) = 8 \div 5 = 1.6$
- $r(6) = 13 \div 8 = 1.625$
- $r(7) = 21 \div 13 = 1.6125$

These figures eventually converge on the Golden Ratio, 1.61803398, an irrational number that never repeats. According to Persaud's, "the Golden Ratio is helpful since it also has a visual meaning that aids in the creation of naturally balanced and aesthetically beautiful shapes and designs" [4].

The science underlying a great smile

For centuries, the golden ratio has been utilized in building, art, and dentistry. In art, proportion describes how various elements' sizes relate to one another within a composition. It guides the viewer's recognition of forms by ensuring visual harmony, realism, and balance. Artists can produce realistic portraits, exaggerate effects, or communicate symbolic meaning by carefully manipulating proportions. Trigonometry, algebra, and geometry are all essential to architectural design. These mathematical formulas are used by architects to plan their preliminary sketch designs. In order to make it more user-friendly, they also compute the likelihood of problems the construction team can encounter when realizing the design goal in three dimensions. The application of golden proportion to dentistry was recorded by Levin E.I. [1] in 1978. He was the

first to define the relationship between proportion, an attractive smile, and dentition that is, the growth and arrangement of the teeth in the mouth and to explain its story and mathematics.

According to the Golden Ratio, 1.618:1.0 is the optimal ratio between longer and shorter lengths that are desired for aesthetic appeal. It is regarded as the benchmark for flawless beauty and is the most crucial instrument for assessing symmetry and superiority. Lombardi was the first to propose the use of the Golden Proportion in dentistry in 1973 [3]. This means that he first proposed the existence of naturally proportionate teeth but rejected the idea of using the Golden Proportion to create aesthetically pleasing teeth due to its limitations. Because he saw that every tooth, from the central incisor to the first premolar, had a repeating ratio. Levin noted that "both the lateral incisor's width and the canine's width were in Golden Proportion to the maxillary central incisor's width." According to him, "the lateral incisor's visible width is 62% (0.618) of the central incisor, and the canine's is 62% (0.618) of the lateral incisor."

In dentistry, the golden percentage is used to assess the proportion of teeth. The creation of flawless anterior teeth and attractive smiles is the primary objective of restorative dentistry. The whole appearance of the patient's face, including their lips, smile line, and facial shape, determines this, Successful restorative dentistry is defined using a wide spectrum of research, beliefs, and techniques. One such idea that has been highlighted numerous times in the dentistry literature is the golden percentage. However, in practice, the ratio's usefulness in producing proportionate designs is constrained by the inherent variations in patients' tooth arrangement and look. According to the RED (Recurring Esthetic Dental) proportion, as you move distally (away from the center), the ratio between the widths of succeeding maxillary teeth (from the front) should remain constant. For instance, the ratio of a canine's width to the lateral incisor's width should be equal to the ratio of a lateral incisor's width to a central incisor.

In order to arrange treatment, Digital Smile Design (DSD) software uses mathematical principles like as the Golden Proportion and facial analysis utilizing reference lines. This results in a balanced and aesthetically beautiful smile that compliments a patient's characteristics and makes them happy. Present day digital tools help the dental professionals to plan treatment more accurate, get more consistent outcomes and patient involved process which means give the chance to the patient to take part in design their smile. These tools aim to do this by creating digital blueprints of the patients' teeth that includes exact measurements of the tooth width, shape [2].

Mathematical ratios in aesthetic dentisry

Golden Ratio

Let's look into what's the golden ratio? It's a number or proportion that we can find in everything which is balanced like in nature, art and science. Especially in our teeth. It's equal to 1.618 and indicated by phi (∅).

Mathematical Definition

Let's look into how it works out in mathematics. Two parts are divided from a straight line and a is Longer part and b as a Shorter part,

$$\frac{a}{b} = \frac{a+b}{a}$$

$$= \emptyset \approx 1.618$$

- $\boldsymbol{a} \rightarrow \textit{length of the longer segment}$
- $\boldsymbol{b} \rightarrow \textit{length of the shorter segment}$
- $\boldsymbol{a} + \boldsymbol{b} \rightarrow \textit{total length of the line}$

So, the ratio between the full line with the longer part is the same as the ratio of the longer part with the shorter part. Let's focus on how this works for dentistry, the golden ratio helps in the digital smile line by determining the tooth width and shape for pleasing smiles and appearance. Let's say, "the lateral incisor has a width of 1 unit, then the central incisor should be 1.618 unit ideally."

The Golden Percentage

"The Golden Percentage was calculated by dividing the width of each maxillary central incisor, lateral incisor, and canine by the frontal inter-canine width and multiplying the resulting value by 100 for each tooth. The obtained values were rounded off to the nearest whole number. If the values from canine to canine were $10\%, 15\%, 25\%, 25\%, 15\%, and\ 10\%$, it indicated that the six maxillary anterior teeth were in the Golden Percentage" [1].

Appropriate proportion to have a beautiful smile

As previously said, there is no universally accepted concept of beauty. It depends on a range of factors, such as cultural norms and individual preferences. What individuals find attractive can varies widely from one another people. However, we have developed a consistent mathematical model to direct the design of front teeth (anterior dental aesthetics) following meticulous measurements and consultations with dental specialists. This methodology helps create a symmetrical and appealing smile by using the Golden Ratio ($\emptyset$) to determine optimal tooth widths and a certain ratio that relates tooth width to height.

We believe that a gummy grin, in which the teeth are overshadowed by an excessive amount of pink gum tissue, is often not seen as beautiful. For this reason, the height of the Teeth is crucial to the aesthetics of a smile. We examined research on tooth breadth in the preceding section [1]. We now focus on tooth height while keeping width in mind. A tooth's height that adds to a lovely grin depends on a number of variables, such as its width, the lips' shape, and even local or cultural standards of beauty. When a tooth's height and width match a certain ratio, it looks its best. In light of our findings, we propose that the optimal height (H) to width (W) ratio for the front teeth (anterior teeth) should be around $\left(\frac{7}{3}\right) or\ \approx 2.333$.

Let $W_c, W_l\ and\ W_k$ stand for the widths and $H_c, H_l\ and\ H_k$ for the heights of the canine, lateral incisor, and maxillary central incisor, respectively. Based on the $\frac{7}{3}$ ratio, the suggested height $H_c \approx 2.33x$ if $W_c = x$. In order to maintain vertical symmetry, we postulate that the $\frac{7}{3}$ ratio applies uniformly to all anterior teeth, ensuring that:

1. Intra-tooth Dimensions: Each anterior tooth's height and width must be in a 7/3 ratio:

$$\frac{H_c}{W_c} \approx \frac{H_l}{W_l} \approx \frac{H_k}{W_k} \approx \frac{7}{3}$$

2. Inter-tooth Width Proportions: The widths ought to follow the Golden Ratio. ($\phi \approx 1.618$)

$$\frac{W_c}{W_l} \approx \phi \; and \; \frac{W_l}{W_k} \approx \phi$$

The second condition implies that $W_l \approx 0.618\, W_c$ $(which\ is\ close\ to\ 0.62x)$ and $W_k \approx 0.382\, W_c$ $(which\ is\ close\ to\ 0.38x)$.

We establish the relationship between the heights of neighboring teeth by combining these two proportional conditions. When the width ratios are substituted into the height ratios, the following results are obtained, when these are combined, ϕ also determines the height proportions:

$$\frac{H_c}{H_l} \approx \frac{\left(\frac{7}{3}\right).W_c}{\left(\frac{7}{3}\right).W_l} \approx \frac{W_c}{W_l} \approx \phi$$

This calculation demonstrates that upholding the $\frac{7}{3}$. All three anterior teeth have the same width-to-height ratio requires that these teeth's heights likewise adhere to the Golden Ratio sequence:

$$\frac{H_c}{H_l} \approx \phi \; and \; \frac{H_l}{H_k} \approx \phi$$

This mathematical conclusion proves that the Golden Proportion must control the anterior segment's breadth and height in order to achieve the best possible aesthetic outcome. This two-dimensional Golden Proportion model is now visually shown in the accompanying figure 1.

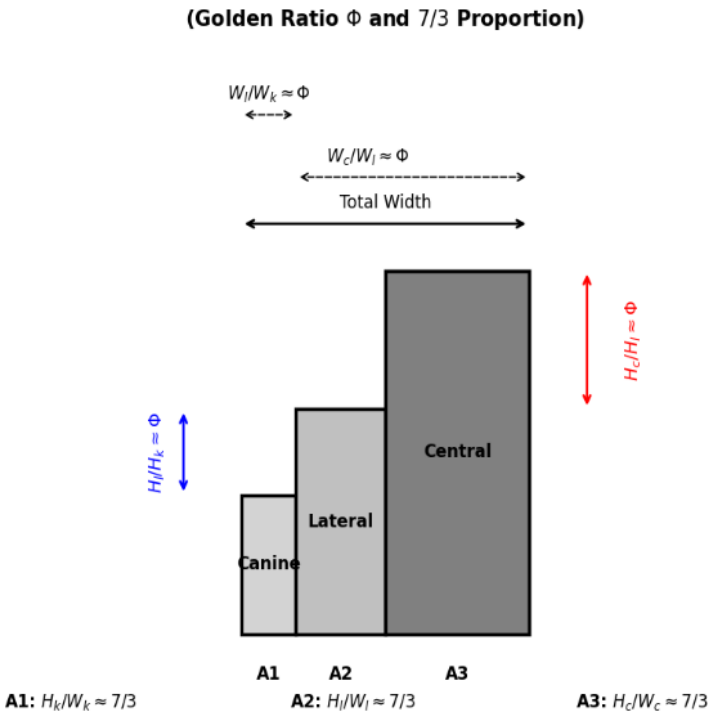

Figure 1: Anterior dental aesthetics: A unified mathematical model

According to the investigation, in order to prevent a "gummy smile" and obtain the perfect smile curve, the proportion $\frac{7}{3}$ must be applied to each of the three anterior teeth's height-to-width ratios, making sure that both heights and widths are in Golden Proportion. Gingivectomy, also known as crown length surgery or gum lift, is a cosmetic dentistry operation that shapes the gum tissue in order to improve the smile and reveal more teeth.

Results and Discussion

Our aim is finding the width-to-height $\frac{H}{W}$ ratio is the target. The vertical Golden proportion ϕ was successfully applied utilizing a two-phase geometric constraint mechanism.

1. **Intra-Tooth Proportional Constant:** The optimal intra-tooth ratio was determined to be the constant $\frac{H}{W} \approx \frac{7}{3} \ (or\ 2.333)$. by analyzing morphological data and aesthetic preferences.

2. **Inter-Tooth Proportional Goal:** The width norm from the literature was upheld, necessitating**,**

$$\frac{W_c}{W_l} \approx \phi \ and \ \frac{W_l}{W_k} \approx \phi$$

Given that both teeth have the same $\frac{H}{W}$ constant, the ratio reduces to the width ratio directly:

$$\frac{H_c}{H_l} \approx \frac{W_c}{W_l} \approx \phi$$

For the grin to have the best possible geometric harmony, the heights of the neighboring anterior teeth must likewise be in the Golden Proportion order, according to this derived relationship. Therefore, the aesthetic problem is no longer a set of criteria instead of a single geometric necessity. New aesthetic models, developed by Levin (1978) [1], addressed “proportional ratios as a one-dimensional problem with emphasis primarily on the horizontal plane (width). The height dimension was never mathematically linked to the basic Golden Ratio, despite the fact that the clinical value of tooth height was acknowledged (especially in regulating gingival zenith).” This theoretical gap is closed by our discovery that the optimization of the width-to-height ratio to the $\frac{7}{3}$ constant mathematically assures ϕ symmetry in the vertical dimension.

Conclusion

A person's grin line and curve are influenced by the size and shape of their lips, face muscles, teeth, and gums. Because the human body has the "Golden Ratio," culture and individual tastes can influence what is deemed beautiful. However, the mathematical analysis of this study yields a single, quantifiable blueprint for perfect aesthetic symmetry. Nonetheless, this study's mathematical analysis produces a single, measurable blueprint for optimal visual symmetry.

The main finding is that effective use of the Golden Ratio $(\emptyset)$ and the $\frac{7}{3}$ proportion $\left(\frac{H_c}{H_l} \approx 2.333\right)$ is necessary for a unified anterior grin. It is mathematically demonstrated that the

heights of the canine, lateral, and central teeth fall within the Golden Proportion sequences $\frac{H_c}{H_l} \approx \phi \; and \; \frac{H_l}{H_k} \approx \phi$ with respect to the ratio of height-to-width. This unified standard guarantees that cosmetic have a complete geometric goal for restorative and surgical operations because both horizontal and vertical symmetry are simultaneously improved.

Additionally, the use of mathematical ratios in cosmetic dentistry provides a "crucial component of accuracy and objectivity to the creation of gorgeous smiles. These proportions enable dental professionals to go beyond just subjective evaluations by offering quantifiable standards that may result in more reliable and scientifically proven outcomes." The Golden Ratio continues to be a reliable foundation for assessing facial and dental harmony among many concepts. However, for completely natural results, each patient's distinct facial anatomy, ethnicity, and aesthetic preferences demand specific personalization. It should therefore continue to be adaptable.

Although this study establishes a theoretical basis, future research should focus on confirming the $\frac{7}{3}$ constant through morphometric analysis of real dental datasets, digital modeling, and clinician feedback. By elucidating its geometric or biological roots, this ratio's scientific basis will be reinforced. By combining 3D simulations with CAD/CAM smile design technology, the dependability of this proportional system in clinical practice may be further examined. Additionally, examining cultural influences on aesthetic perception and anatomical variability will assist ensure that the model is adaptable and applicable to a variety of populations.

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